Chapter 18
SWAPS 2

B - Decomposition & Combination

Currency Swaps

- Also called *Cross currency swaps* (XCCY).
- The legs of the swap are denominated in **different currencies**.
- Currency swaps change the **profile of cash flows**.
- Many possibilities for the CF exchanges: fixed-fixed, fixed-floating *(Circus swap)* & floating-floating (*XCCY basis swap*).
- Reference rates were **IBOR**, usually USD LIBOR, Euribor (EUR IBOR), JPY TIBOR. They have been **replaced** by **SOFR** & Ameribor (USD), **€STR** or Euro Short-Term Rate (EUR), **TONAR** (JPY), etc.

Example:

Situation: ExxonMobil has USD debt, but wants to increase EUR debt.

Solution: A swap.

ExxonMobil pays EUR. A Swap Dealer pays USD.

Example (continuation):

ExxonMobil pays EUR. A Swap Dealer pays USD.

• <u>Swap Details</u>:

- ExxonMobil pays **3.5%** in EUR, with a Notional principal: **EUR 20 M**
- Swap Dealer pays **4%** in USD, with a Notional principal **USD 26 M**
- Frequency of payments = 6-mo (**s.a.**)
- Duration = 4 years
- S_t = 1.30 USD/EUR.

Every six month, Exxon pays **EUR 350,000** & receives **USD 520,000**.

Note that Exxon and SD have implicitly fixed S_t for 4 years at:

S_{t+j}= **USD 520,000**/**EUR 350,000** = 1.485714 USD/EUR j=6,12,.,48. ¶

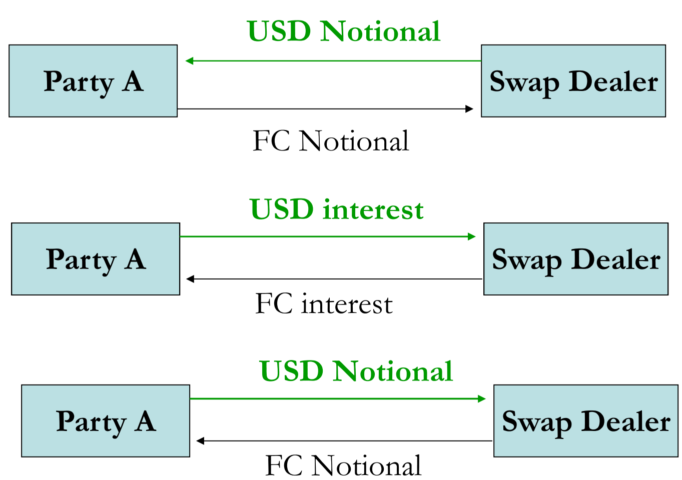
<u>Usual CFs in a XCCY swap</u>

In currency swaps the notional principals are **usually exchanged**. There are three sets of cash flows:

• At inception

• Periodic interest payments

• At end of term



<u>Note</u>: Similar to an exchange of bonds. At inception, Party A receives USD "funding" and pays interest during the duration ("tenor") of swap.

Currency Swaps: Variations

Key: Both legs are different currencies. **Different Instruments**:

1. Fixed-Fixed

Example: Exxon-Mobile example.

2. Fixed-Floating (also called *Circus swap* = Combined Interest Rate & Currency Swap)

Example: IBM pays 3-mo Ameribor in USD and receives 5% in EUR. ¶

3. Floating-Floating (also called *cross currency basis swap,* if initial exchange of notionals occurs)

Example: IBM pays 3-mo Ameribor in USD and receives 3-mo ESTR – **30 bps**. This EUR/USD XCCY swap is quoted "**-30 bps.**"

Note: **-30 bps** is the *spread* in EUR. The *spread* could be zero (IRP holds), positive or negative. ¶

Coss-currency Basis Swaps

The **difference** between the **two floating rates** in a currency swap is called the *basis swap spread*, usually quoted against USD Ameribor (unsecured) flat. For example, in the IBM example above:

EUR/USD *basis swap spread* = (3-mo ESTR – **30 bps**) – (3-mo Ameribor)

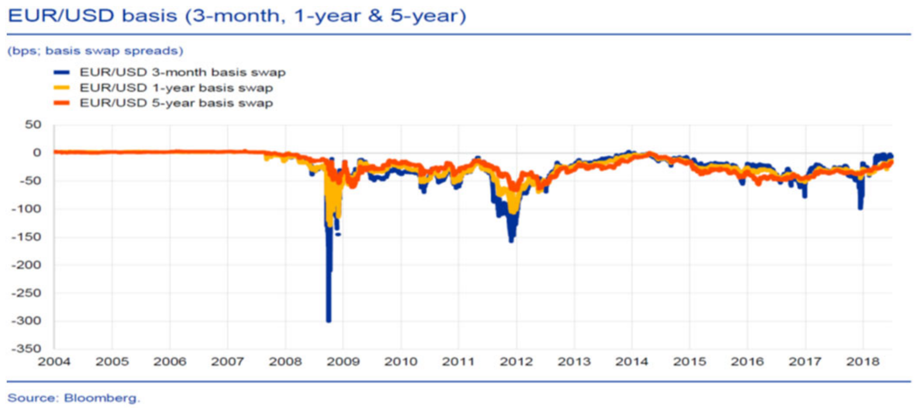
Note: Theoretically, IRP arbitrage should ensure that a XCCY basis swap trades without a spread.

• Who participates in a XCCY swap?

- The EUR/USD XCCY is used by **European banks to fund USD** assets if other USD funding sources become inaccessible. The typical **other side** of this swap are **European** issuers (in particular, agencies, international bodies, and sovereigns), which swap USD debt issues into EUR.

- European firms **issue USD bonds and swap proceeds into EUR** to **diversify** into other funding sources and, potentially, get **cheaper** funding.

• *Euro-USD Basis Swaps Spreads*: **2004-2019**



• After the financial crisis, the EUR-USD *XCCY basis swap spread* became negative. That is, a party seeking to switch from EUR to USD holdings is required to pay a premium for borrowing USD in the XCCY swap market).

The XCCY spread is taken as an indicator of funding conditions. For example, during the 2008-2009 period there was a shortage of USD.

Valuation of Currency Swaps

A currency swap can be decomposed into a position in two bonds:

- A domestic bond (or foreign currency 1 bond)
- A foreign bond (or foreign currency 2 bond)

V = Value of Swap (to DC payer) = NPV of FC bond – NPV of DC bond

In previous example the swap value to ExxonMobil is:

$$\mathbf{V} = B_D - S_t\, B_F$$

B_F: Value of FC denominated bond underlying the swap.

B_D: Value of DC denominated bond underlying the swap.

S_t: Spot exchange rate.

Note: For the Swap Dealer, the swap value (in DC) is:

$$\mathbf{V} = S_t\, B_F - B_D$$

Example FI:

A U.S. financial institution (FI) is involved in a currency swap:

Terms:

- Frequency of payment: Annual.
- Notional principals: **DKK 53 million** & **USD 10 million**.
- FI get **5.5% p.a.** in DKK against **6% p.a.** in USD
- Payments: FI pays **USD 0.6M** & receives **DKK 2.915M**
- Swap will last for another three years (T = 3 years).

S_t = **0.18868 USD/DKK**.

Term structure in Denmark & U.S. is *flat,* with i_{DKK} = 5% & i_{USA} = 6.5%.

$$\text{FI} \underset{\text{DKK 2.915M (5.5\%)}}{\overset{\text{USD 0.6M (6\%)}}{\rightleftarrows}} \text{Swap Dealer}$$

Note: At maturity, the principals will also be exchanged.

Example FI (continuation):

$$\text{FI} \underset{\text{DKK 2.915M (5.5\%)}}{\overset{\text{USD 0.6M (6\%)}}{\rightleftarrows}} \text{Swap Dealer}$$

These periodic exchanges set an implicit forward contract (the swap forward rate), fixing S_t for each year at:

S_{t+j} = USD 0.6M / DKK 2.915M = **0.2058319 USD/DKK**, j =1, 2, 3.

There are 3 *long* forward contract for FI:

Maturity (T) = 1 year, 2 years, & 3 years.

Size: **DKK 2.915M**

Swap forward FX rate: **0.2058319 USD/DKK**

Note: Potentially, like in all long forward contracts, the FI profits if, at any T, the market forward rate, $\boldsymbol{F_{t,t_j}}$, is higher than **0.2058319 USD/DKK**.

Example FI (continuation):

Discount rates: $i_{DKK} = 5\%$ & $i_{USA} = 6.5\%$.

Coupons: **DKK 2.915M** & **USD 0.6M**

T = 3 years.

S_t = **0.18868 USD/DKK.**

$$\text{FI} \xrightarrow{\text{USD 0.6M (6\%)}} \text{Swap Dealer}$$

$$\text{FI} \xleftarrow[\text{DKK 2.915M (5.5\%)}]{} \text{Swap Dealer}$$

$$B_D = \frac{.6M}{(1+.065)} + \frac{.6M}{(1+.065)^2} + \frac{.6M}{(1+.065)^3} + \frac{10M}{(1+.065)^3} = \text{USD } 9{,}867{,}577$$

$$B_F = \frac{2.915M}{(1+.05)} + \frac{2.915M}{(1+.05)^2} + \frac{2.915M}{(1+.05)^3} + \frac{55.915M}{(1+.05)^3} = \text{DKK } 53{,}721{,}661$$

$V_{US\ FI}$ = (53,721,661) * (.18868) – 9,867,577 = **USD 268,585.45**.

V_{SD} (paying DKK and receiving USD) = USD -268,585.45. ¶

Decomposition into Forward Contracts

The CFs of currency swap can be **valued** as a **series of forward contracts**, which are set by the exchanges of interest payments & principals.

Recall the value of a long forward contract is the present value of the amount by which the forward price exceeds the delivery price.

Example FI (continuation):

Annual exchanges: **DKK 2,915,000** = **USD 600,000**

At maturity, final exchange: **DKK 53 M** = **USD 10 M**

⇒ Each of these payments represents an implicit forward contract.

- Swap forward rate fixed by the annual exchanges of interest payments:
USD 0.6M/ **DKK 2,915,000** = **0.2058319 USD/DKK**.

- Swap forward rate fixed by the last exchange of principals at T = 3 years:
USD 10M/ **DKK 53M** = **0.1886792 USD/DKK**. ¶

• We value the swap forward rate relative to the IRPT forward rate, $\boldsymbol{F_{t,T}}$:

$$\boldsymbol{F_{t,T}} = S_t * \frac{\left(1 + i_d * \frac{T}{360}\right)}{\left(1 + i_f * \frac{T}{360}\right)}$$

Suppose in the swap, we are long the FC (the FI is long DKK). Then, the PV, using i_d as the discount rate, of each annual payment j is:

$$(\boldsymbol{F_{t,t_j}} - \text{Swap forward rate at time } t_j) * \frac{\text{Amount of FC}}{(1 + i_{d,j})^{t_j}}$$

Example FI (continuation):

FI's value of the exchange of principals at $T = 3$ years ($\text{Value}_{\text{FI,Principals}}$).

$$\boldsymbol{F_{t,T=3-yr}} = \textbf{.18868 USD/DKK} * \frac{(1 + .065)^3}{(1 + .05)^3} = \textbf{.19688 USD/DKK}$$

Swap forward rate = **USD 10M**/**DKK 53M** = **0.1886792 USD/DKK**.

$$\text{Value}_{\text{FI,Principals}} = (\textbf{.19688} - \textbf{0.1886792}) * \frac{\textbf{53M}}{(1 + .065)^3} = \textbf{USD 0.35982M}$$

Note: We can do the same for each exchange of CFs. ¶

• Alternatively, we can value the CFs in terms of forward DC.

Notation:

t_j: time of the jth settlement date

$i_{d,j}$: domestic interest rate applicable to time t_j

$\boldsymbol{F_{t,t_j}}$: forward exchange rate applicable to time t_j, calculated by IRPT.

• PV to the FI of the swap forward contract set by the corresponding exchange of payments at time t_j:

$$(\textbf{DKK 2,915,000} * \boldsymbol{F_{t,t_j}} - \textbf{USD 0.6M}) * \frac{1}{(1 + i_{d,j})^{t_j}}$$

• PV to the FI of the swap forward contract set by the exchange of principal payments at time T:

$$(\textbf{DKK 53M} * \boldsymbol{F_{t,T}} - \textbf{USD 10M}) * \frac{1}{(1 + i_{d,T})^{T}}.$$

⇒ The value of a currency swap can be calculated from the term structure of forward rates and the term structure of $i_{d,j}$.

Example (continuation): Reconsider FI Example.

S_t = .18868 USD/DKK.

i_{DKK} = 5%

i_{USA} = 6.5%.

Using IRPT, the one-, two- and three-year forward exchange rates are:

$$F_{t,T=1\text{-yr}} = .18868 \text{ USD/DKK} * \frac{(1+.065)}{(1+.05)} = .19137 \text{ USD/DKK}$$

$$F_{t,T=2\text{-yr}} = .18868 \text{ USD/DKK} * \frac{(1+.065)^2}{(1+.05)^2} = .19411 \text{ USD/DKK}$$

$$F_{t,T=3\text{-yr}} = .18868 \text{ USD/DKK} * \frac{(1+.065)^3}{(1+.05)^3} = .19688 \text{ USD/DKK}$$

Example (continuation): Reconsider FI Example.

• The value of the implicit swap forward contracts corresponding to the exchange of interest are therefore (in millions of USD):

$$(\text{DKK } 2.915 * .19137 \text{ USD/DKK} - \text{USD } .6) * \frac{1}{(1+.065)} = \text{USD } -.03957\text{M}$$

$$(\text{DKK } 2.915 * .19411 \text{ USD/DKK} - \text{USD } .6) * \frac{1}{(1+.065)^2} = \text{USD } -.03013\text{M}$$

$$(\text{DKK } 2.915 * .19688 \text{ USD/DKK} - \text{USD } .6) * \frac{1}{(1+.065)^3} = \text{USD } -.02160\text{M}$$

• The final exchange of principal involves receiving **DKK 53M** & paying USD 10M. The value of the forward contract is:

$$(\text{DKK } 53\text{M} * .19688 \text{ USD/DKK} - \text{USD } 10\text{M}) * \frac{1}{(1+.065)^3} = \text{USD } 359{,}816$$

• Then, the total value of the swap is (in USD):

359,816 – 39,570 – 30,130 – 21,600 = **USD 268,516**.

⇒ FI would be willing to sell this swap for **USD 268,516**. ¶

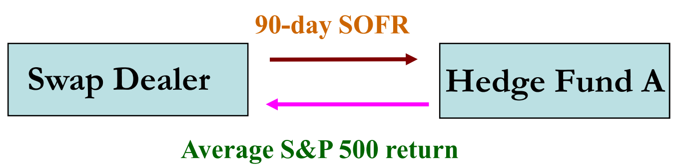
Equity Swaps

Equity swaps have two legs: One of the legs is pegged to the return of a stock or market index. Usually, the other is pegged to a floating rate (SOFR or ESTR **plus or minus a spread**).

Terms include notional principal, duration and frequency of payments.

Example: Stock returns against a floating rate.

On April 1, Hedge Fund A enters into a 3-year equity swap. Every quarter, Hedge Fund A pays the average S&P 500 return in exchange of 90-day SOFR (count 30/360) .



Example: (continuation)

Notional principal = **USD 40 million**.

Data at inception (April 1):

S&P500 index = 4100

90-day SOFR = **3%**.

On July 1, Hedge Fund A will pay (or receive if sum is negative):

USD 40 M * [**S&P 500 return (04/01 to 07/01)** – **0.03** * 90/360].

If on July 1, S&P 500 = 4153 ⇒ **Return** = 4153/4100 – 1 = **.0130**.

Then the payment will be:

USD 40M * [**.0130** – **0.03** * 90/360] = **USD 0.22M**.

On July 1, SOFR is set for the next 90-day period (07/01 to 10/01). ¶

• **Variations**

- Equity return against a **fixed rate** (S&P500 against 2%)
- Equity return against **another equity return** (S&P500 against NASDAQ)
- Equity return against a **foreign equity return** (S&P500 against FTSE)
- Equity swaps **with changing notional** ("reinvested") principals

• **Q: Why equity swaps?**

(1) Avoid **transaction costs** and **taxes**.
(2) Avoid **legal limits** (margins, capital controls) and **institutional rules**.
(3) Keep equity positions (and voting shares) **without equity risk**.

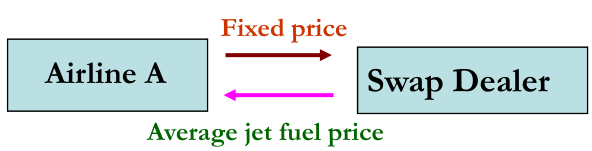
Commodity Swaps

Commodity swaps work like any other swap: one legs involves a fixed commodity price and the other leg a (variable) commodity market price.

Unlike futures commodity contracts, *cash settlement* is the norm.

Example: Jet fuel oil swap.

Airline A enters into a 2-year jet-fuel oil swap. Every quarter, Airline A receives the average market price –based on a known price quote- & pays a fixed price.



Example: (continuation)

Cash settlement: If the average jet-fuel price paid is above (below) the fixed price, the SD will repay (receive from) the airline the difference in what it paid versus the fixed price. ¶

Note: There is no futures contract for jet fuel oil. A swap **completes** the **market**.

You can consider the 2-year swap as a **collection** of 8 **forward contracts**.

• Q: Why commodity swaps?

(1) *A commodity swap eliminates basis risk*

Southwest Airlines has used NYMEX crude oil and heating oil futures contracts to hedge jet fuel price risk. But, this introduces basis risk.

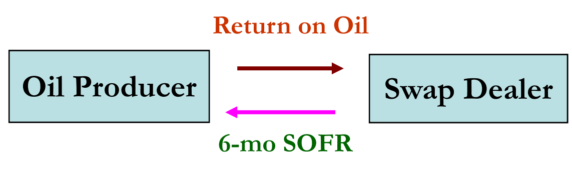
(2) *Expanded market*

Since there is cash settlement, market participants do not need to have the infrastructure to take delivery.

• Commodity for interest swap

They work like an equity swap: One leg pays a return on a commodity, the other leg pays an interest rate (say, SOFR plus or minus a spread).

Example: An oil producer enters into a 2-year swap. Every six month, the oil producer pays the return on oil –based on NYMEX Light Crude Oil– and receives 6-mo SOFR.



• Valuation of Commodity Swaps

Commodity swaps are valued as a series of **commodity forwards**, each priced at inception with zero value.

The fixed coupon payment is a weighted average of commodity forward prices.

Credit Default Swaps (CDS)

• A CDS is an agreement between two parties. One party buys *protection against specific risks associated with credit events* –i.e., defaults, bankruptcy, restructuring, or credit rating downgrades. Cash settlement is allowed.

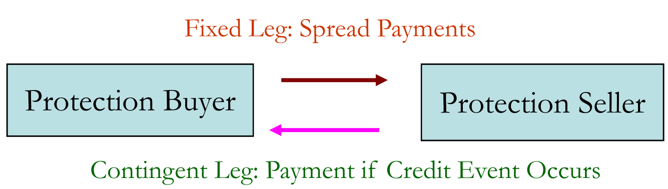
• Facts:

- Today, CDS is the most widely traded credit derivative product.
- Outstanding amount: **USD 9.3 trillion** (November 2022).
- Maturities range from 1 to 10 years (5 years is the most common).
- Most CDS's are in the USD 10M to 20M range.

• CDS contracts are governed by the International Swaps and Derivatives Association (**ISDA**), which provides standardized definitions of CDS terms, including definitions of what constitutes a credit event.

CDS Mechanism

- One party *buys protection* –i.e., "sells" risk or "short credit exposure- and the counterparty *sells protection* –i.e., "buys" risk or credit exposure.
- Protection buyer pays a **periodic fee** (the *spread*) to protection seller.
- Protection seller agrees to pay the protection buyer a set amount **if there is a credit event** (usually, default).
- Usually, collateral rules apply to seller; following a **5-day 99% VaR**.

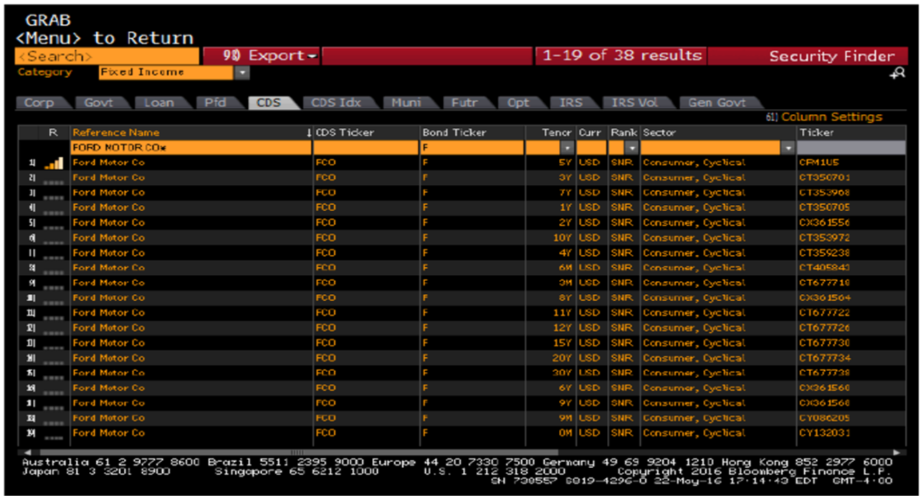


Though it is not necessary to buy a CDS, the protection buyer **tends to own the underlying asset** subject to risk.

Note: The spread is positively related to the likelihood of credit event.

CDS Quotes

Below we show a snapshot from a *Bloomberg* terminal (from window for "Par CDS spread"). Ford has multiple CDS contracts outstanding, each based on a different bond. The first one, is a CDS based *on the 5-year senior bond* (the most liquid CDS contract).



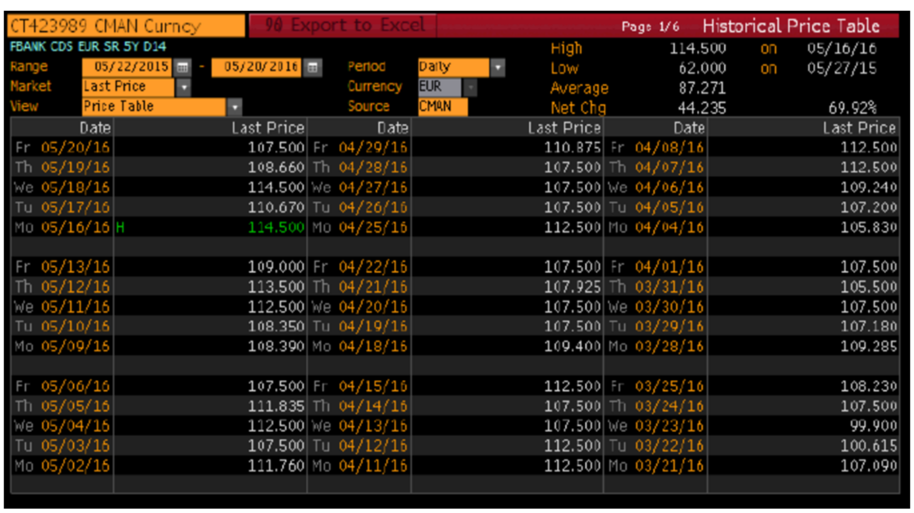
GRAB
<Menu> to Return
<Search> | 98) Export | 1-19 of 38 results | Security Finder
Category: Fixed Income
Corp | Govt | Loan | Pfd | CDS | CDS Idx | Muni | Futr | Opt | IRS | IRS Vol | Gen Govt
61) Column Settings

R	Reference Name	CDS Ticker	Bond Ticker	Tenor	Curr	Rank	Sector	Ticker
	FORD MOTOR CO*		F					
1)	Ford Motor Co	FCO	F	5Y	USD	SNR	Consumer, Cyclical	CFM1U5
2)	Ford Motor Co	FCO	F	3Y	USD	SNR	Consumer, Cyclical	CT350701
3)	Ford Motor Co	FCO	F	7Y	USD	SNR	Consumer, Cyclical	CT353968
4)	Ford Motor Co	FCO	F	1Y	USD	SNR	Consumer, Cyclical	CT350705
5)	Ford Motor Co	FCO	F	2Y	USD	SNR	Consumer, Cyclical	CX361556
6)	Ford Motor Co	FCO	F	10Y	USD	SNR	Consumer, Cyclical	CT353972
7)	Ford Motor Co	FCO	F	4Y	USD	SNR	Consumer, Cyclical	CT359238
8)	Ford Motor Co	FCO	F	6M	USD	SNR	Consumer, Cyclical	CT405843
9)	Ford Motor Co	FCO	F	3M	USD	SNR	Consumer, Cyclical	CT677718
10)	Ford Motor Co	FCO	F	8Y	USD	SNR	Consumer, Cyclical	CX361564
11)	Ford Motor Co	FCO	F	11Y	USD	SNR	Consumer, Cyclical	CT677722
12)	Ford Motor Co	FCO	F	12Y	USD	SNR	Consumer, Cyclical	CT677726
13)	Ford Motor Co	FCO	F	15Y	USD	SNR	Consumer, Cyclical	CT677730
14)	Ford Motor Co	FCO	F	20Y	USD	SNR	Consumer, Cyclical	CT677734
15)	Ford Motor Co	FCO	F	30Y	USD	SNR	Consumer, Cyclical	CT677738
16)	Ford Motor Co	FCO	F	6Y	USD	SNR	Consumer, Cyclical	CX361560
17)	Ford Motor Co	FCO	F	9Y	USD	SNR	Consumer, Cyclical	CX361568
18)	Ford Motor Co	FCO	F	9M	USD	SNR	Consumer, Cyclical	CY080205
19)	Ford Motor Co	FCO	F	0M	USD	SNR	Consumer, Cyclical	CY132031

Australia 61 2 9777 8600 Brazil 5511 2395 9000 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2016 Bloomberg Finance L.P.
SN 730557 G019-4296-0 22-May-16 17:14:43 EDT GMT-4:00

CDS Quotes

Below we show another snapshot from a *Bloomberg* terminal, showing historical prices (=CDS *spreads*). We show the last price of each day. CDS spreads do vary.



CT423989 CMAN Curncy | 98 Export to Excel | Page 1/6 Historical Price Table
FBANK CDS EUR SR 5Y D14
Range 05/22/2015 - 05/20/2016 | Period Daily | High 114.500 on 05/16/16
Market Last Price | Currency EUR | Low 62.000 on 05/27/15
View Price Table | Source CMAN | Average 87.271
Net Chg 44.235 69.92%

Date	Last Price	Date	Last Price	Date	Last Price
Fr 05/20/16	107.500	Fr 04/29/16	110.875	Fr 04/08/16	112.500
Th 05/19/16	108.660	Th 04/28/16	107.500	Th 04/07/16	112.500
We 05/18/16	114.500	We 04/27/16	107.500	We 04/06/16	109.240
Tu 05/17/16	110.670	Tu 04/26/16	107.500	Tu 04/05/16	107.200
Mo 05/16/16 H	114.500	Mo 04/25/16	112.500	Mo 04/04/16	105.830
Fr 05/13/16	109.000	Fr 04/22/16	107.500	Fr 04/01/16	107.500
Th 05/12/16	113.500	Th 04/21/16	107.925	Th 03/31/16	105.500
We 05/11/16	112.500	We 04/20/16	107.500	We 03/30/16	107.500
Tu 05/10/16	108.350	Tu 04/19/16	107.500	Tu 03/29/16	107.180
Mo 05/09/16	108.390	Mo 04/18/16	109.400	Mo 03/28/16	109.285
Fr 05/06/16	107.500	Fr 04/15/16	112.500	Fr 03/25/16	108.230
Th 05/05/16	111.835	Th 04/14/16	107.500	Th 03/24/16	107.500
We 05/04/16	112.500	We 04/13/16	107.500	We 03/23/16	99.900
Tu 05/03/16	107.500	Tu 04/12/16	112.500	Tu 03/22/16	100.615
Mo 05/02/16	111.760	Mo 04/11/16	112.500	Mo 03/21/16	107.090

CDS Benefits

Besides **hedging event risk**, the CDS provides the following benefits:

- A short positioning vehicle that does not require an initial cash outlay.
- Access to maturity exposures not available in the cash market.
- Access to credit risk not available in the cash market due to a limited supply of the underlying bonds.
- Investments in foreign credits without currency risk.
- Ability to effectively 'exit' credit positions in periods of low liquidity.

CDS: Not Insurance

- In car insurance, you need to own the car and show damage to receive compensation from a claim. In a CDS contract, the protection buyer does **not need to own the underlying** credit exposure.
- Protection seller is **not necessarily regulated**. No reserves are required.
- CDS's are **mark-to-market** (in the US).

Typical CDS Quote

A 5-year CDS quote for Bertoni Bank (on April 17, 2015)

Notional amount = **USD 10 million** (= Czech Rep Eurobond holdings)

Premium or Spread: **160 bps** (related to risk of Czech Republic)

Maturity: 5 years

Frequency: Quarterly Payments

Credit event: Default

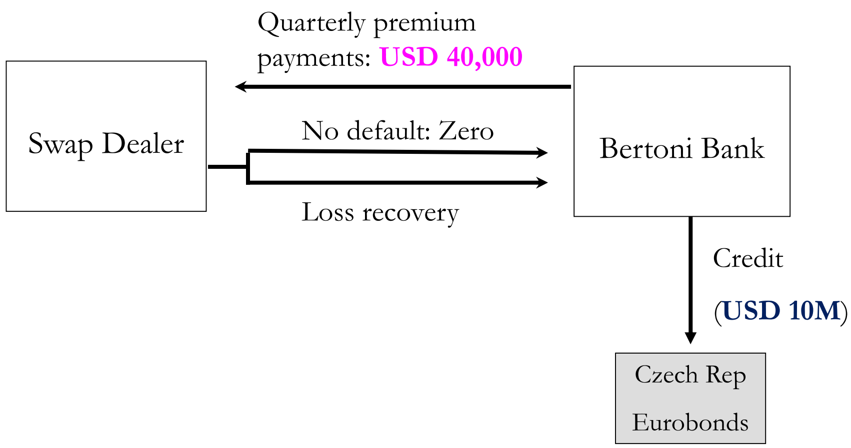
• Calculation of the Spread

Q; How much Bertoni Bank pays for protection?

(0.0160/4) * **USD 10M** = **USD 40,000** *(every quarter as a premium for protection against company default)*

If the Czech Republic (Eurobond issuer) defaults, the CDS covers the notional **USD 10M**.

Diagram for Bertoni Bank's CDS



Medium risk entity for default

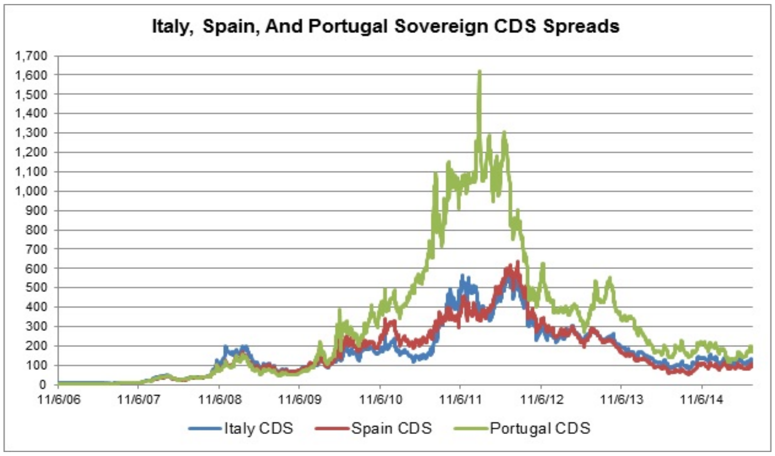
CDS Spread

The spread (C = 160 bps) is related to the default risk of Czech Republic.

Czech Republic Government Bond 10-year YTM: 2009 - 2019

• CDS Spreads Do Reflect Default Risk.

Since we look at CDSs as insurance for bondholders; an increase in CDS premiums indicates that investors are becoming worried about the safety of their investments.



Higher country risk ⇒ Higher CDS premiums

• CDS Spreads Do Reflect Default Risk.

Similar behavior seen in the table (2023) below, from Damodaran (NYU): https://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/ctryprem.html.

Country	*Adj. Default Spread*	*Equity Risk Premium*	*Country Risk Premium*	*Corporate Tax Rate*	Moody's rating
Abu Dhabi	0.60%	6.79%	0.85%	15.00%	Aa2
Albania	5.51%	13.71%	7.77%	15.00%	B1
Algeria	3.68%	11.13%	5.19%	26.00%	NR
Andorra	2.33%	9.23%	3.29%	18.98%	Baa2
Angola	7.95%	17.16%	11.22%	25.00%	B3
Anguilla	7.93%	17.13%	11.19%	25.63%	NR
Antigua & Barbuda	7.93%	17.13%	11.19%	25.63%	NR
Argentina	14.68%	26.65%	20.71%	35.00%	Ca
Armenia	4.40%	12.15%	6.21%	18.00%	Ba3
Australia	0.00%	5.94%	0.00%	30.00%	Aaa
Austria	0.49%	6.63%	0.69%	24.00%	Aa1

CDS: Settlement

• Settlement can either be physical or cash.

(1) *Cash Settlement*

– Protection seller pays buyer **the difference** between **par value** (***N***) & **market price** of a debt obligation of the reference entity (say, market price of similar Czech Republic Eurobonds, ***R***). That is, seller pays:

(100 – ***R***)% of Notional

– To establish value of reference obligation (***R***%), a dealer poll/auction can be used.

(2) *Physical Settlement*

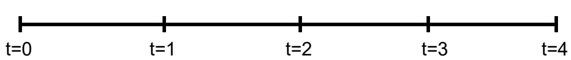
– Protection seller pays buyer **par value** (***N***) & receives an **acceptable obligation** (say, Czech Republic Eurobonds) worth *R,* from buyer.

– Buyer of protection can choose, within certain limits, what obligation to deliver. Puts buyer in a CDB-like situation (pick lower ***R*** possible).

Note: Think of what the seller gets in the event of default as *recovery,* ***R***.

CDS: Pricing

Suppose we want to price a 1-year CDS. Thus, there are 4 events.



Effective date

Nominal amount = ***N***

Premium = C (annualized)

Quarterly payment = ***N*** * (C/4)

There are 5 possible outcomes in this CDS contract:

– No default (4 premium payments are made by bank to investor until the maturity date)

– Default occurs on t_1, t_2, t_3, or t_4

Steps

1) Assign probability to each event –i.e., default at t_1, default at t_2, etc.

2) Calculate PV of payoff for each outcome (assuming δ_i [$=1/(1+r_i)^i$] as the discount rate for period *i*):

 Seller's net default payment is $\boldsymbol{N} * (1 - R)$

 Buyer's payments is $\boldsymbol{N} * C/4$

3) Expected NPV of CDS = Sum of PV of five payoffs multiplied by their probability of occurrence.

 $E[NPV_{CDS}] \approx 0 \Rightarrow$ Determine *fair* C such that $E[NPV_{CDS}] \approx 0$.

Notes: We think of the P_i's as "*survival*" probabilities over an interval:

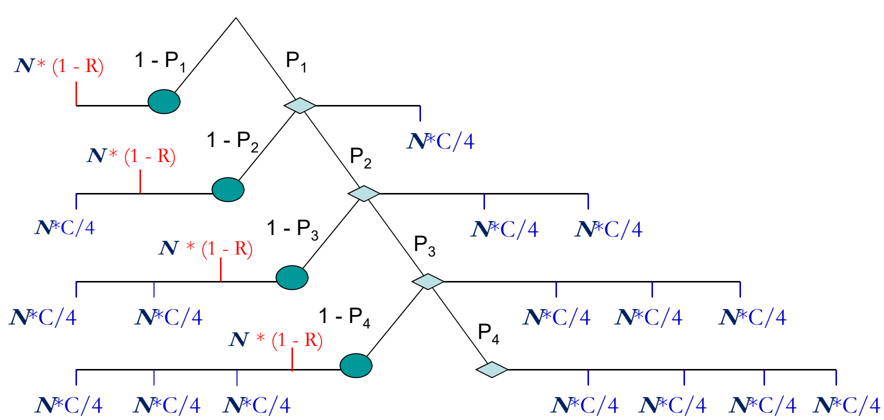
Probabilities $\Rightarrow P_i$ [No default at t_i – issuer still alive at time t_i]

$\Rightarrow 1 - P_i$ [Default at t_i – issuer is "dead" at time t_i]

Technically, P_i is the probability of surviving over interval $[t_i - t_{i-1}]$.

CDS: Payoff Diagram

CFs from the seller's point of view (red=outflows, blue=inflows):



Summary of Events and Payoffs

Description	Premium Payment PV	Default Payment PV	Probability
Default at t_1	0	$N * (1 - R) * \delta_1$	$(1 - P_1)$
Default at t_2	$N * C/4 * \delta_1$	$N * (1 - R) * \delta_2$	$P_1 *(1 - P_2)$
Default at t_3	$N * C/4 * (\delta_1 + \delta_2)$	$N * (1 - R) * \delta_3$	$P_1 * P_2 * (1 - P_3)$
Default at t_4	$N * C/4 * (\delta_1 + \delta_2 + \delta_3)$	$N * (1 - R) * \delta_4$	$P_1 * P_2 * P_3 * (1 - P_4)$
No default	$N * C/4 * (\delta_1 + \delta_2 + \delta_3 + \delta_4)$	0	$P_1 * P_2 * P_3 * P_4$

- $E[NPV_{Seller}]$ = NPV{Premium payments} – NPV{Default payments}

- To calculate the E[NPV of CDS], we need as inputs:
  - Known: *N*, C (determined in the contract)
  - Undetermined/Unknown: $P_i = \{P_1, P_2, P_3, P_4\}$; **R**; and the discount rate for period *i*, $\delta_i = 1/(1 + r_i)^i$.

CDS: Pricing and Inputs

- There are two popular ways to get P_i's:

  (1) **Assume a probability distribution**, for example, the *exponential*. The higher the risk (and spread), the higher the decay in the survival probability.

  (2) **Use no-arbitrage model**. After some assumption, we can use market prices of similar bonds (ideally, from same issuer), B_J, and T-bond prices, B_{RF}, to compute expected PV of cost of default loss as $(B_J - B_{RF})$. Then, we "extract" the implied P_i's. Which one?

- **R** is calculated from **historical** ("average") recovery rates. In many situations, **R** = **40%** is used as the default input. Constant?

- We use $\delta_i = 1/(1+r_i)^i$ from term structure. Under assumptions, we use same discount rate for C/4 & ***N****(1 – **R**). But, we can use different discount rates for the *defaultable* part (***N***) and *non-defaultable* parts (C). Realistic?

CSD: Calculation of PV of CDS and Pricing

Expected Present Value of Credit Default Swap = $E[NPV_{CDS}]$ =

$= (P_1 * P_2 * P_3 * P_4) * [N * C/4 * (\delta_1 + \delta_2 + \delta_3 + \delta_4)]$

$- (1 - P_1) * N * (1 - R) * \delta_1$

$- P_1 * (1 - P_2) * [N * (1 - R) * \delta_2 - N * C/4 * \delta_1]$

$- P_1 * P_2 * (1 - P_3) * [N * (1 - R) * \delta_3 - N * C/4 * (\delta_1 + \delta_2)]$

$- P_1 * P_2 * P_3 * (1 - P_4) * [N * (1 - R) * \delta_4 - N * C/4 * (\delta_1 + \delta_2 + \delta_3)]$

Recall that using this formula, we price the CDS –i.e., determine *fair* C. At t = 0, $E[NPV_{CDS}] = 0$ (or, ≈ 0) $\Rightarrow$ get C such that $E[NPV] \approx 0$.

$N * (1 - R) * \{(1 - P_1)*\delta_1 + P_1*(1 - P_2)*\delta_2 + P_1*P_2*(1 - P_3)*\delta_3 + P_1*P_2*P_3*(1 - P_4)*\delta_4\}$

$= N * C/4 * \{P_1*(1 - P_2)*\delta_1 + P_1*P_2*(1 - P_3)*(\delta_1 + \delta_2) + P_1*P_2*P_3*(1 - P_4)*(\delta_1+\delta_2+\delta_3)$

$+ (P_1*P_2*P_3*P_4)*(\delta_1+\delta_2+\delta_3+\delta_4)\}$

CSD: Calculation of PV of CDS and Pricing

$N*(1 - R) * \{(1 - P_1)*\delta_1 + P_1*(1 - P_2)*\delta_2 + P_1*P_2*(1 - P_3)*\delta_3 + P_1*P_2*P_3*(1 - P_4)*\delta_4\}$

$= N*C/4 * \{ P_1*(1 - P_2)*\delta_1 + P_1*P_2*(1 - P_3)*(\delta_1 + \delta_2) + P_1*P_2*P_3*(1 - P_4)*(\delta_1+\delta_2+\delta_3) +$
$(P_1*P_2*P_3*P_4)*(\delta_1+\delta_2+\delta_3+\delta_4)\}$

Then,

$$C = \frac{4 * (1 - R) * \{(1 - P_1)/P_1*\delta_1 + (1 - P_2)*\delta_2 + P_2*(1-P_3)*\delta_3 + P_2*P_3*(1 - P_4)*\delta_4\}}{\{(1-P_2)*\delta_1 + P_2*(1-P_3)*(\delta_1+\delta_2) + P_2*P_3*(1-P_4)*(\delta_1+\delta_2+\delta_3) + (P_2*P_3*P_4)*(\delta_1+\delta_2+\delta_3+\delta_4)\}}$$

$\Rightarrow$ **C** is the *fair CDS spread.*

Example: Expected NPV for Bertoni Bank's CDS, with **2 payments left**

Notional amount = **USD 10 million** (Czech Republic Eurobonds)

Premium or Spread = **C** = **160 bps**

Maturity: 6 months (2 payments left)

Frequency: Quarterly Payments

Credit event: Default

Discount rates: 3-mo = **0.035**; & 6-mo = **0.037**.

Recovery Rate = (1 – **R**) = 60%

Quarterly payments = N * **C**/4 = **USD 10M** * (**0.0160**/4) = **USD 40,000**

Probability of Default: P_1 = .99; & P_2 = .985.

E[NPV of Credit Default Swap (in USD M)] =

$= (.99 * .985) * [.040 * \{1/(1+ .035/4)^1 + 1/(1+ .037/4)^2\}]$

$– (1 – .99) * 10 * .60 * 1/(1+ .035/4)^1$

$– .99 * .015 * [10 * .60 * 1/(1+ .037/4)^2 – .040 * 1/(1+ .035/4)^1] = -0.0694$

Note: Like swaps, at inception the PV of CDS ≈ 0. In this case, we call the spread (or premium) *fair*.

Example (continuation): Pricing CDS

Today, we want to price a similar CDS to the Bertoni Bank's CDS. Then, we set **C** such that E[NPV] ≈ 0. That is, if a similar CDS is issued today with 2 payments left, the *fair spread* is: **C** = **303.19 bps**, since:

E[NPV of Credit Default Swap (in USD M)] =

$= (.99 * .985) * [.075796 * \{1/(1+ .035/4)^1 + 1/(1+ .037/4)^2\}]$

$– (1 – .99) * 10 * .60 * 1/(1+ .035/4)^1$

$– .99 * .015 * [10 * .60 * 1/(1+ .037/4)^2 – .075796 * 1/(1+ .035/4)^1]$

$\approx 0.$

Then, the quarterly payments are:

= N * **C**/4 = **USD 10M** * (**0.030319**/4) = **USD 75,796**. ¶

CDS: Risks

• The main risk is *counterparty risk* –i.e., the seller defaults. If a major counterparty (say, AIG, Lehman) defaults a large number of market participants are left un-hedged.

• If a large seller defaults, network domino effects are possible.

• Collateral & margin can spiral out of control. Asset values are correlated with CDS protection sold & the economy. To post more collateral, firms have to de-leverage (sell assets at worst time: *fire sale*.)

• Modeling CDS spreads is complicated:
- Market is illiquid –i.e., difficult to trust observed market prices.
- P_i's are not easy to determine.
- Fat-tailed and left-skew distributions.
- Difficult to aggregate risks (hard to measure default correlations).

CDS: Summary

• CDS are bilateral contracts, often sold and resold among parties.

• Large market, due to netting, the notional size of the CDS market is approximately 1/10th the size of the gross notional market.

• Due to its protection nature CDS market represents over one-half of the global credit derivative market.

• CDS allows a party who buys protection to trade and manage credit risks in much the same way as market risks.

Combination of Swaps

• Recall that swaps change the profile of cash flows.
• Swaps solve problems: *Financial Engineering*.

Example: A Brazilian oil producer is exposed to two forms of price risk:

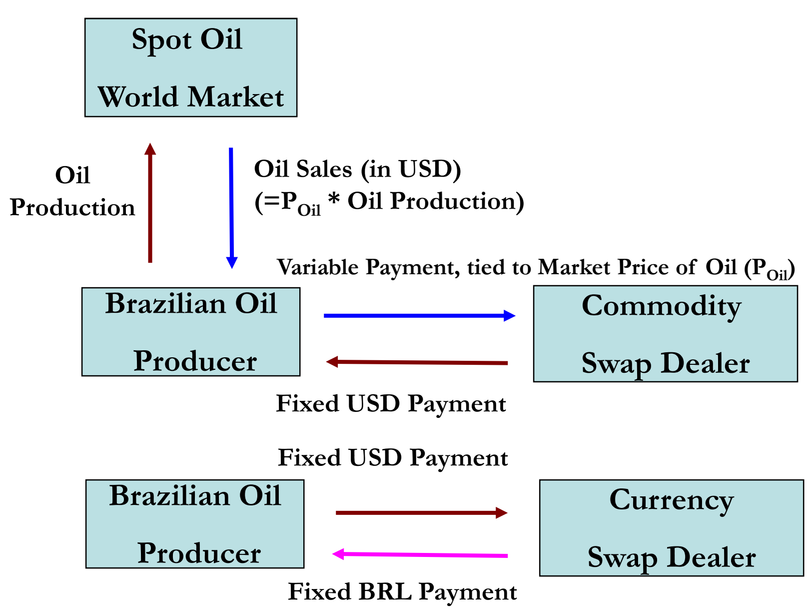
- P_{Oil} (priced in USD/barrel of oil) ⇒ commodity price risk.
- S_t (BRL/USD) ⇒ FX risk.

Situation: Since expenses are in BRL, the Brazilian oil producer wants to **fix P_{Oil} in BRL/barrel of oil**.

Solution: *Financial Engineering*, a combination of swaps can do it!

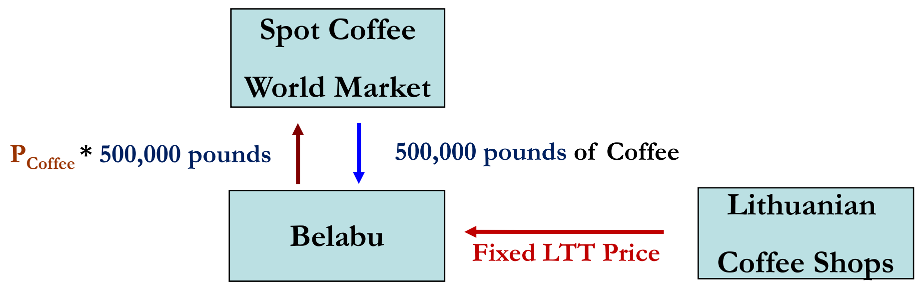
Note: This is a typical problem for commodity producers and buyers from non-USD zones: Commodities are priced in USD.

Diagram: Structured Solution for Brazilian Oil Producer



Example: Combining Swaps.

- Belabu, a Lithuanian coffee roaster, uses **500,000 pounds** of Colombian coffee every 6-months. Colombian coffee trades spot at $\mathbf{P_{Coffee}}$ (USD).
- Belabu has contracts to sell its output at a fixed price for **4 years**.



- $\mathbf{S_t}$ = **4.74 LTT/USD**
- $\mathbf{P_{Coffee}}$ = 1.95 USD/pound.
- Goal: Belabu wants to **fix the price of coffee in LTT/pound**.

  ⇒ Belabu, simultaneously, enters **three swaps**: A commodity swap, an interest rate swap, & a currency swap.

(1) Commodity swap dealer:

Belabu receives: Average market price of coffee ($\mathbf{P_{Coffee}}$).

Belabu pays: A fixed-price of **USD 2.05 per pound**

Current mid-price quote for a 4-yr coffee swap is **USD 1.99 per pound**. (Dealer subtracts/adds **USD .06** to its mid-price.)

(2) Interest rate swap dealer:

Belabu receives: USD fixed rate amount

Belabu pays: USD floating rate amount

4-yr swap interest rate quote: **8.2%** against 6-mo. SOFR.

(3) Currency swap dealer:

Belabu receives: a USD floating rate amount

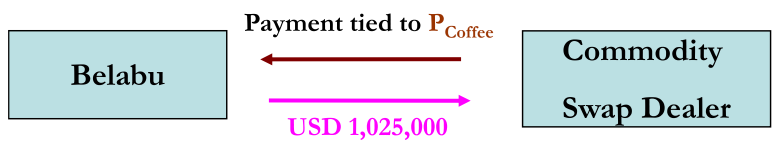
Belabu pays: a LTT fixed-rate amount

4-year LTT-for-USD currency swap quote: **7.8%** against 6-mo. SOFR.

Details

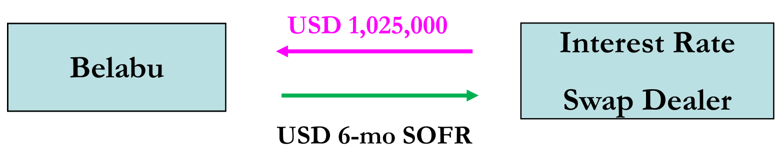
1. Determine the number of USD Belabu will need every six months:

500,000 pounds * **USD 2.05/pound** = **USD 1,025,000**



2. Determine notional principal required on a USD interest rate swap for the fixed-rate side to generate **USD 1,025,000** every 6-mo (**8.2%** rate)

USD 1,025,000 / **.041** = USD 25,000,000



Details (continuation)

3. Calculate the present value of the cash flows on the fixed-rate side of the interest rate swap using the current **8.2%**:

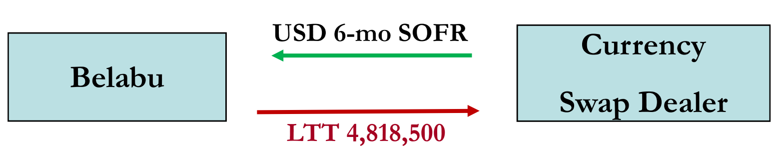
PV(**USD 1,025,000**; **.041**, **8 periods**) = **USD 6,872,600**

4. Translate the PV of the USD cash flows to its LTT equivalent:

4.74 LTT/USD * **USD 6,872,600** = **LTT 32,576,124**

5. Determine the LTT CFs on the fixed-rate side of the LTT-for-USD currency swap having a NPV of **LTT 32,576,124** at **7.8%** current rate.

Coupon(PV = **LTT 32,576,124**; **.039**, **8 periods**) = **LTT 4,818,500**



Details (continuation)

6. Determine the LTT notional principal that would generate the semiannual payments of **LTT 4,818,500** at **7.8%:**

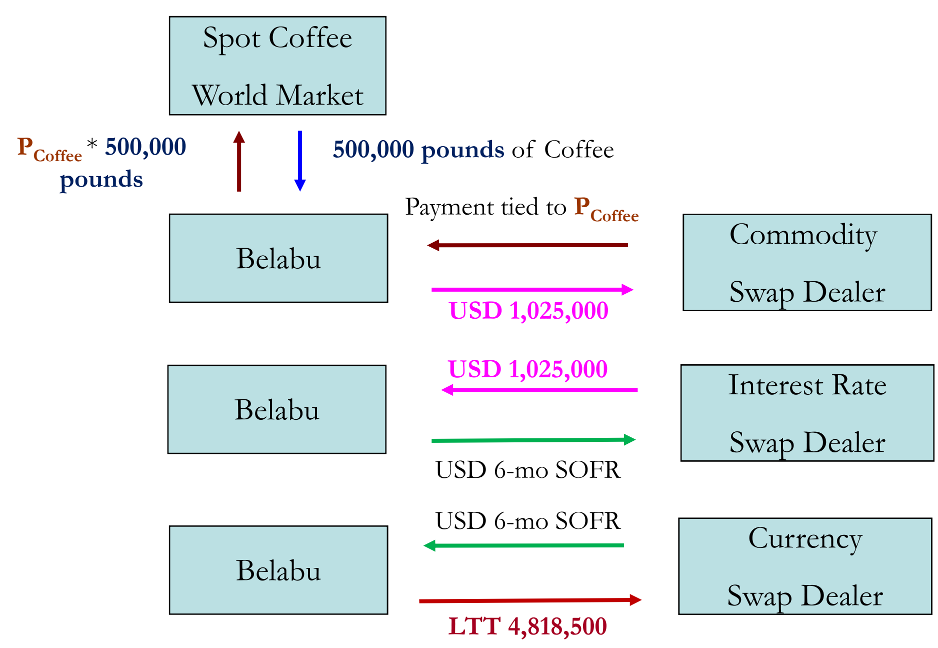
LTT 4,818,500 / **.039** = LTT 123,551,282.10

⇒ The structured solution has fixed the price of coffee for four years:

LTT 4,818,500 / **500,000 pounds** = **9.637 LTT/pound of coffee**.

That is, Belabu pays **9.637 LTT/pound of coffee** for 4 years.

Diagram: Structured Solution for Belabu



Synthetic Instruments

Synthetic instruments are not securities at all.

- CF streams formed by combining the CF streams from one set of instruments to replicate the CF streams of another set of instruments.
- When combined with appropriate cash positions, it is possible to use swaps to replicate the CF stream associated with virtually any instrument.

Example: Dual currency bond.

Situation: Bertoni Bank issued dual currency bonds for **USD 1M**.

Coupon payments: JPY 6.5 M.

Frequency of payments: semiannual.

Maturity: 5 years.

Features: Sold and redeemed in USD. Interest payments in JPY.

S_t = 100 JPY/USD

CF for BB: At issue (t = 0): BB receives **USD 1M**.

Every 6-mo: BB pays **JPY 6.5 M**

At maturity (t = 5): BB pays **USD 1M**.

• Bertoni Bank's CF can be synthesized using:

- A Corporate USD straight bond.
- A fixed-for-fixed currency swap.

Example (continuation):

• Instruments:

- 7.5% Chase Bonds with (at least) 5 years to maturity.
- A currency SD offers **USD 7.5%** against **JPY 6.5%** (SD pays USD)

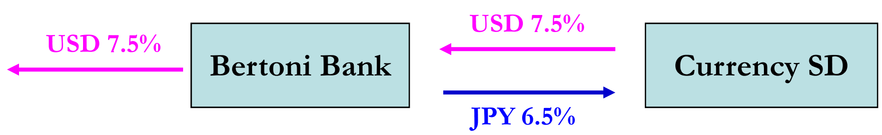
• At t = 0, BB sells short **USD 1M** worth of Chase bonds

⇒ BB receives **USD 1M** and makes **USD 7.5%** coupon payments.

• BB also enters a fixed-for-fixed currency swap & make JPY payments.

⇒ BB pays **JPY 6.5%** & receives **USD 7.5%.**

• At t = 5, BB buys back the Chase bonds for **USD 1M**.



Example: Synthetic Equity

Situation:

Goyco Corp. has **USD 3M** to invest for two years.

Goyco is bullish on the Japanese market in the near future.

Goyco decides to invest in synthetic Japanese equity using as tools:

- A fixed-rate note
- An equity swap.

• Goyco buys a 7.8% FV = **USD 3M**, 2-yr bond, trading at par.

• At the same time, Goyco enters into a 2-yr equity swap:

- Goyco pays the swap dealer **6.5%** annually and receives the return on the Nikkei 225 (r_{Nikkei}).
- Goyco pays the swap dealer when $r_{Nikkei} < 0$.
  (in addition to the fixed-rate payment of **6.5%**.)
- Notional: **USD 3 million**

Example (continuation):

- Goyco's position has a return equal to the Nikkei 225 plus **130 bps**.
- Net effect: Creation of a (synthetic) equity position for Goyco.

⇒ In some countries, swaps are **off-balance sheet** items: Goyco only shows its 2-year corporate bond on its balance sheet.